A Model of Political Competition with Activists applied to the elections of 1989 and 1995 in Argentina.

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July 10, 2006

Abstract

The mean voter theorem suggests that all parties should rationally converge to the electoral center. Typically this leads to an outcome which is unattractive to the rich. This paper develops a general stochastic model of elections in which the electoral response is affected by the valence (or quality) of the candidates. Contributions made by policy-motivated activists can influence valence, leading to the failure of the mean voter theorem. The model is then applied to the presidential elections in 1989 and 1995 in Argentina, to suggest why Carlos Menem, who won in 1989 with a populist platform, was able to win in 1995 with quite different policies that favored the rich.

JEL Classification C13, C72, D72.

Key words: Local Nash Equilibrium, Stochastic Electoral Model, Valence.
1 INTRODUCTION

The main contenders in Argentina’s 1989 presidential election were Carlos Menem, the candidate of the PJ (Partido Justicialista) and Eduardo Angeloz, the candidate of the UCR (Union Civica Radical).

Angeloz had the disadvantage of coming from the same political party as the president in office, who decided to contest the election because of hyperinflation. Angeloz’s platform was located in the center-right of the economic axis of the political space. His featured proposal was the so-called “red pen,” namely the downsizing of the national state to pursue a better fiscal performance.

Menem was a charismatic, populist candidate, but lacking a sound political platform. His platform included a universal rise in salaries (salariazó) and a big push to the productive sector (revolucion productiva). This platform, clearly located in the left of the economic axis, gave Menem broad support from the working class, and constituted the key to his electoral victory.

Surprisingly, once in office Menem announced policies that were the opposite of his electoral promises, including the liberalization of trade, the privatization of several state companies, the freeze of public salaries and the deregulation of the markets. Also, in 1991 Menem established a currency board, the so-called “Convertibility Plan,” which succeeded in controlling hyperinflation and thus provided the basis for four years of macroeconomic stability and growth.

However, the Convertibility Plan proved to be vulnerable to both exogenous “contagion” and fiscal imbalances and led to a progressive appreciation of the Argentinean currency. Such appreciation generated losers and winners. Among the latter were the recently privatized firms, (seeking to maximize the value of their assets and profits denominated in dollars) and most of the upper middle class, who came to enjoy the benefits of inexpensive imported goods. The losers consisted of the export-oriented sector, together with many small and medium-sized firms and their workers, who could not survive the appreciation of the peso and the opening of the economy.

Menem went for re-election in 1995, with a promise to maintain the administration’s, economic policy. Because he had broken the electoral promises of 1989, Menem lost about 15% of the leftist votes. However, because of the high standard of living achieved during Menem’s administration, the upper middle class compensated for the working class deflection. Menem’s share of the vote in 1995 was approximately 35% from the traditional PJ constituency (working class) and 15% from other constituenc-
cies, mainly voters who had previously chosen the UCR (Gervasoni 1997). The 1995 election resulted in the re-election of Menem (with almost 50% of the votes), coupled with a very good performance from a new party, the FREPASO (29%), and a resounding defeat for the UCR (17%).

This real-world story is at odds with the electoral models used to analyze elections. First, the position of the main parties’ platforms in the policy space in 1989 contradicts the “mean voter theorem.” This theorem predicts the convergence of the candidates to the electoral center. Second, it is surprising that Menem’s policies benefited a very different constituency from the one that had brought him to office in 1989.

The aim of this paper is as follows. First, we aim to explain the apparent paradox that actual political systems generally display divergence rather than convergence. The case of the elections in Argentina in the 1989 and 1995 presidential elections is a good example. Second, we aim to understand the nature “political realignment” (Sundquist 1973) in general, and the particular realignment that we argue occurred between these elections in Argentina.

A key idea is that the convergence result does not necessarily hold if there is an asymmetry in the electoral perception of the “quality” of party leaders (Stokes 1992). The average weight given to the perceived quality of the leader of the $j^{th}$ party is called the party’s valence. In empirical models, a party’s valence is usually assumed to be independent of the party’s position. The addition of valence typically contributes to the statistical significance of the model. This has been indicated by the Bayes’ factors in models of elections for four countries (Schofield and Sened 2006). In general, valence reflects the overall degree to which the party is perceived to have shown itself able to govern effectively in the past, or is likely to be able to govern well in the future (Penn 2003). Formal models of elections incorporating valence have been developed recently (Ansolabehere and Snyder 2000; Aragones and Palfrey 2002; Groseclose 2001), but the theoretical results to date have been somewhat inconclusive.

By introducing the assumption that there are two kinds of valence, this paper offers a general model of elections. The first kind of valence is a fixed or exogenous valence for a party. The exogenous valence of party $j$ is denoted $\lambda_j$ and, as in empirical models, is assumed to be independent of the party’s position. The second kind is known as activist valence. When party $j$ adopts a policy position, $z_j$, then the activist valence of the party is denoted $\mu_j(z_j)$. Implicitly, we build on a model originally due to Aldrich (1983a,b) and Aldrich and McGinnis (1989), in which activists provide crucial resources of time and money to their chosen party. The party then uses these resources
to enhance its image before the electorate, thus improving its valence. It is worth emphasizing that although activist valence is affected by party position, it does not operate in the usual way, namely by influencing voter choice through the distance between a voter’s preferred policy position, say $x_i$, and the party position. Instead, we assume that as party $j$’s activist support, $\mu_j(z_j)$, increases, then, ceteris paribus, all voters become more likely to support party $j$ over all other parties. Nonetheless, because activists are likely to be more extreme than the typical voter, by choosing a policy position that maximizes activist support, party $j$ loses centrist voters. This forces the party to calculate the marginal condition that maximizes vote share. We offer a theorem that gives this marginal condition as a (first order) balance condition. Because activist support is in terms of some fixed resource, we shall assume that the activist function exhibits decreasing returns to scale. In formal terms, we assume that the functions themselves are concave, so that the Hessians of the activist functions are negative-definite.

We consider the Argentinean case a especially challenging test for the implications of a general electoral model. To the best of our knowledge, no other polity has suffered such deep transformations in such a short period. Indeed, between 1989-1995, Argentina’s polity experienced: (i) the saliency of a new dimension, namely the value of its currency, (ii) a sharp change in the population’s perception of the relative “quality” of the two major parties, the PJ and the UCR, and (iii) the emergence of a potent activist group, in the form of the recently privatized firms and their political allies. Because the depth and speed of these transformations gives us clues about of the strategic behavior of the same political actors in different situations, we regard the Argentinean polity in the 1990’s to be a crucial test of the model.

Although our model is static by nature, the theorem and corollaries presented below allow for comparative analysis over a range of parameters, including dimensionality, voter’s average perception of the relative quality of the candidates, and the number of parties. We explain the resulting transformations in Argentina’s polity by using various theorems associated with different aspects of the model.

First, we cite a theorem that asserts that when the activist functions are sufficiently concave, then a Nash equilibrium exists. The party positions in this equilibrium will depend on the parameters of the model. The theorem is applied to an examination of how a party’s equilibrium position will be affected when it responds to different activists groups with contradictory agenda. As the intensity of support from a group of activists increases, a candidate will consider the benefits of moving along a “balance locus”
between them and another group of activists. Under the premise that parties act strategically, the party’s move can cause responses from the other parties on any dimension. Under certain circumstances, a political “realignment” will occur.

The paper is structured as follows. Sections 2 and 3 present the formal model. (The proofs of the two theorems are in a companion paper.) Section 4 uses the activist vote maximizing model in an informal fashion to discuss interesting aspects of Argentina’s electoral process in the 1990’s. Section 5 concludes.

2 Local Nash Equilibrium with Activists and Vote Maximizing Parties

The electoral model presented here is an extension of the multiparty stochastic model of McKelvey and Patty (2006), modified by inducing asymmetries in terms of valence. The justification for developing the model in this way is the extensive empirical evidence that valence is a natural way to model the judgements made by voters of party leaders (Schofield and Sened 2006). There are a number of possible choices for the appropriate model for multi-party competition. The simplest one, which is used here, is that the utility function for party $j$, is proportional to the vote share, $V_j$, of the party. With this assumption, we can examine the conditions on the parameters of the stochastic model which are necessary for the existence of a pure strategy Nash equilibrium (PNE). Because the vote share functions are differentiable, we use calculus techniques to obtain conditions for positions to be locally optimal. Thus we examine what we call local pure strategy Nash equilibria (LNE). From the definitions of these equilibria it follows that a PNE must be a LNE, but not conversely. A necessary condition for an LNE is thus a necessary condition for a PNE. A sufficient condition for an LNE is not a sufficient condition for PNE. Indeed, additional conditions of concavity or quasi-concavity are required to guarantee existence of PNE.

The key idea underlying the formal model is that party leaders attempt to estimate the electoral effects of party declarations, or manifestos, and choose their own positions as best responses to other party declarations, in order to maximize their own vote share. The stochastic model essentially assumes that party leaders cannot predict vote response precisely, but can estimate an expected vote share. In the model with valence, the stochastic element is associated with the weight given by each voter, $i$, to the average perceived quality or valence of the party leader.
Definition 1. The Stochastic Vote Model $M(\lambda, \mu, \beta; \Psi)$ with Activist Valence.

The data of the spatial model is a distribution, $\{x_i \in X\}_{i \in N}$, of voter ideal points for the members of the electorate, $N$, of size $n$. We assume that $X$ is a compact convex subset of Euclidean space, $\mathbb{R}^w$, with $w$ finite. Without loss of generality, we adopt coordinate axes so that $\tfrac{1}{n}\sum x_i = 0$. By assumption $0 \in X$, and this point is termed the electoral origin. Each of the parties in the set $P = \{1, \ldots, j, \ldots, p\}$ chooses a policy, $z_j \in X$, to declare. Let $z = (z_1, \ldots, z_p) \in X^p$ be a typical vector of party policy positions.

Given $z$, each voter, $i$, is described by a vector

$$u_i(x_i, z) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p)),$$

where

$$u_{ij}(x_i, z_j) = \lambda_j + \mu_j(z_j) - \beta \|x_i - z_j\|^2 + \epsilon_j = u_{ij}^*(x_i, z_j) + \epsilon_j. \quad (1)$$

Here $u_{ij}^*(x_i, z_j)$ is the observable component of utility. The term, $\lambda_j$, is the fixed or exogenous valence of party $j$, while the function $\mu_j(z_j)$ is the component of valence generated by activist contributions to agent $j$. The term $\beta$ is a positive constant, called the spatial parameter, giving the importance of policy difference defined in terms of a metric induced from the Euclidean norm, $\| \cdot \|$, on $X$. The vector $\epsilon = (\epsilon_1, \ldots, \epsilon_j, \ldots, \epsilon_p)$ is the stochastic error, whose multivariate cumulative distribution will be denoted by $\Psi$.

It is assumed that the exogenous valence vector

$$\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p) \text{ satisfies } \lambda_p \geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1.$$

Voter behavior is modeled by a probability vector. The probability that a voter $i$ chooses party $j$ at the vector $z$ is

$$\rho_{ij}(z) = \Pr[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l), \text{ for all } l \neq j]. \quad (2)$$

$$= \Pr[\epsilon_l - \epsilon_j < u_{lj}^*(x_i, z_j) - u_{lj}^*(x_i, z_j), \text{ for all } l \neq j]. \quad (3)$$

Here $\Pr$ stands for the probability operator generated by the distribution assumption on $\epsilon$. The expected vote share of agent $j$ is

$$V_j(z) = \frac{1}{n} \sum_{i \in N} \rho_{ij}(z). \quad (4)$$

The differentiable function $V : X^p \to \mathbb{R}^p$ is called the party profile function.

Definition 2. Equilibrium Concepts.
(i) A strategy vector $\mathbf{z}^*=(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) \in X^p$ is a local strict Nash equilibrium (LSNE) for the profile function $V : X^p \rightarrow \mathbb{R}^p$ iff, for each agent $j \in P$, there is a neighborhood $X_j$ of $z_j^*$ in $X$ such that

$$V_j(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) > V_j(z_1^*, \ldots, z_j, \ldots, z_p^*)$$

for all $z_j \in X_j \setminus \{z_j^*\}$.

(ii) A strategy vector $\mathbf{z}^*=(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*)$ is a weak Nash equilibrium (PNE) iff, for each agent $j$,

$$V_j(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) \geq V_j(z_1^*, \ldots, z_j, \ldots, z_p^*)$$

for all $z_j \in X$. □

We also use the term local weak Nash equilibrium (LNE) for a local equilibrium where the strict inequality of (i) is replaced with a weak inequality.

The most common assumption in empirical analyses is that $\Psi$ is the Type I extreme value distribution (sometimes called log Weibull). The theorem in this paper is based on this assumption. This distribution assumption is the basis for much empirical work based on multinomial logit estimation (Dow and Endersby 2004).

**Definition 3. The Type I Extreme Value Distribution, $\Psi$.**

The cumulative distribution, $\Psi$, has the closed form

$$\Psi(h) = \exp[-\exp[-h]],$$

with probability density function

$$\psi(h) = \exp[-h] \exp[-\exp[-h]]$$

and variance $\frac{1}{6} \pi^2$. □

It follows from (2) that for each voter $i$, and party $j$, the probability that a voter $i$ chooses party $j$ at the vector $\mathbf{z}$ is

$$\rho_{ij}(\mathbf{z}) = \frac{\exp[u_{ij}^*(x_i, z_j)]}{\sum_{k=1}^p \exp u_{ik}^*(x_i, z_k)}.$$ (5)

See Train (2003). Thus

$$\rho_{ij}(\mathbf{z}) = [1 + \sum_{k=\neq j} \exp(f_k)]^{-1}$$ (6)

where $f_k = \lambda_k + \mu_k(z_k) - \lambda_j - \mu_j(z_j) + \beta||x_i - z_j||^2 - \beta||x_i - z_k||^2$.

We can show that the first order condition for $\mathbf{z}^*$ to be a LSNE is that it be a balance solution.
Definition 4. The balance solution for the model $M(\lambda, \mu, \beta; \Psi)$.

Let $[\rho_{ij}(z)] = [\rho_{ij}]$ be the $n$ by $p$ matrix of voter probabilities at the vector $z$, and let

$$[\alpha_{ij}] = \left[ \frac{\rho_{ij} - \rho^2_{ij}}{\sum_i (\rho_{ij} - \rho^2_{ij})} \right]$$

be the $n$ by $p$ matrix of weighting coefficients.

The balance equation for $z^*_j$ is given by expression

$$z^*_j = \frac{1}{2\beta} \frac{d\mu_j}{dz_j} + \sum_{i=1}^{n} \alpha_{ij} x_i.$$  \hfill (8)

The vector $\sum \alpha_{ij} x_i$ is a convex combination of the set of voter ideal points. This vector is called the weighted electoral mean for party $j$. Define

$$\frac{d\mathcal{E}_j^*}{dz_j} \equiv \sum_{i} \alpha_{ij} x_i.$$ \hfill (9)

Then the balance equation can then be expressed as

$$\left[ \frac{d\mathcal{E}_j^*}{dz_j} - z^*_j \right] + \frac{1}{2\beta} \frac{d\mu_j}{dz_j} = 0.$$ \hfill (10)

The bracketed term on the left of this expression is termed the marginal electoral pull of party $j$ and is a gradient vector pointing towards the weighted electoral mean of the party. This weighted electoral mean is that point where the electoral pull is zero. The vector $\frac{d\mu_j}{dz_j}$ is called the marginal activist pull for party $j$.

If $z^*$ satisfies the balance equation for all $j$, then call $z^*$ the balance solution. \hfill $\square$

The following theorem is proved in Schofield (2006a).

**Theorem 1.** Consider the electoral model $M(\lambda, \mu, \beta; \Psi)$ based on the Type I extreme value distribution, and including both exogenous and activist valences.

(i) The first order condition for $z^*$ to be an LSNE is that it is a balance solution.

(ii) If all activist valence functions are highly concave, in the sense of having negative eigenvalues of sufficiently great modulus, then the balance solution will be a PNE. \hfill $\square$
In the case that the activist valence functions are identically zero, we write the model as \( M(\lambda, \beta; \Psi) \). Then the mean voter theorem for this stochastic model asserts that the “joint mean vector” \( z_0 = (x^*, \ldots, x^*) \) is a PNE, where \( x^* = \frac{1}{n} \sum x_i \). A key step in a proof of this assertion is to determine whether the electoral mean is a LSNE. Using Theorem 1 it is readily shown that when the activist valence functions are identically zero, then for each fixed \( j \), all \( \alpha_{ij} \) are identical. Thus, when there is only exogenous valence, the balance solution satisfies \( z_j^* = \frac{1}{n} \sum x_i \), for all \( j \). By the change of coordinates mentioned above, we can choose \( \frac{1}{n} \sum x_i = 0 \), the electoral origin. In this case, the marginal electoral pull is zero at the origin and the joint origin \( z_0 = (0, \ldots, 0) \) satisfies the first order condition. Schofield (2006b) shows that in this case there is a “convergence coefficient,” \( c \), defined in terms of the exogenous valence differences, the spatial coefficient, \( \beta \), and the electoral variance. The necessary convergence condition for \( z_0 \) to be an LNE is that \( c \leq w \), where \( w \) is the dimension of the policy space. When this condition fails, so does the mean voter theorem. Moreover, concavity of the vote share functions also fails, and there is reason to doubt existence of PNE.

To present this result, we first define \( \nabla_0^* \) to be the \( w \) by \( w \) dimensional electoral covariance matrix about the origin and let \( v^2 \) be the total electoral variance, \( \text{trace}(\nabla_0^*) \). Define the characteristic matrix of the lowest valence party to be

\[
C_1 = 2\beta(1 - 2\rho_1)\nabla_0^* - I
\]

where \( I \) is the identity matrix, and \( \rho_1 \) is the vote share of the lowest valence party when all parties are at the origin. Finally define the convergence coefficient, \( c \), to be \( \text{trace}(2\beta[1 - 2\rho_1]\nabla_0^*) \), so

\[
c = c(\lambda, \beta; \Psi) = 2\beta(1 - 2\rho_1)v^2.
\]

**Theorem 2.** The necessary condition for the joint origin to be a LSNE in the model \( M(\lambda, \beta; \Psi) \) is that the characteristic matrix \( C_1 \) has negative eigenvalues. \( \square \)

Theorem 2 and the following Corollaries are proved in Schofield (2006b).

**Corollary 1.** Consider the model \( M(\lambda, \beta; \Psi) \).

In the case that \( X \) is \( w \)-dimensional, then the necessary condition for the joint origin to be a LNE is that \( c(\lambda, \beta; \Psi) \leq w \). In this case, if there are two parties, with \( \lambda_2 > \lambda_1 \), then the joint origin fails to be a LNE if \( \beta > \beta_0 \) where

\[
\beta_0 = \frac{w[\exp(\lambda_2 - \lambda_1) + 1]}{2v^2[\exp(\lambda_2 - \lambda_1) - 1]}.
\]  \( \square \)
Corollary 2. In the two dimensional case, a sufficient condition for the joint origin to be a LSNE for the model $M(\lambda, \beta; \Psi)$ is that $c(\lambda, \beta; \Psi) < 1$. □

Notice that the case with two parties of equal valence immediately gives a situation with $2\beta(1 - 2\rho_1)v^2 = 0$, irrespective of the other parameters. However, by Corollary 1, if $\lambda_2 \gg \lambda_1$, then the joint origin may fail to be a LNE if $\beta v^2$ is sufficiently large.

When the valence functions $\{\mu_j\}$ are not identically zero, then it is the case that generically $z_0$ cannot satisfy the first order condition. Instead the vector $\frac{d\mu_j}{dz_j}$ “points towards” the position at which the activist valence is maximized. When this marginal or gradient vector, $\frac{d\mu_j}{dz_j}$, is increased (as activists become more willing to contribute to the party) then the equilibrium position is pulled away from the weighted electoral mean of party $j$, and we can say the “activist effect” for the party is increased. If the activist effect is decreased, the local equilibrium of the party is pulled towards the electoral origin. We can say the “electoral effect” is increased.

We now apply these results when there are multiple activist groups.

3 Two-dimensional politics

We consider competition between two parties 1, 2, in a policy space with $w = 2$, where 1 has traditionally been on the left of the economic ($x$) axis, and 2 is on the right of the same axis. To examine the effect of the second dimension ($y$) of policy we develop the model presented by Miller and Schofield (2003) based on “ellipsoidal” utility functions of potential activist groups. In the application to the Argentine polity, the $y – axis$ will represent policy in support of a hard or a soft currency.

Consider the first order equation

$$\frac{d\mu_1}{dz} = 0$$

for maximizing the total valence of 1 when there are two activist groups, L, H, whose preferred points are $L$, $H$, say, and whose utility functions are $u_L$ and $u_H$. The contributions of the groups to party 1 are $\Sigma_L$ and $\Sigma_H$. We make the following set of assumptions.

Assumption 1.

(i) The total activist valence for 1 can be decomposed into two components

$$\mu_1(z_1) = \mu_L(\Sigma_L(z_1)) + \mu_H(\Sigma_H(z_1)).$$

where $\mu_L, \mu_H$ are functions of $\Sigma_L, \Sigma_H$, respectively.
(ii) The contributions $\Sigma_L, \Sigma_H$ can be written as functions of the utilities of the activist groups, so

$$\Sigma_L(z_1) = \Sigma_L(u_L(z_1)) \text{ and } \Sigma_H(z_1) = \Sigma_H(u_H(z_1)).$$  \hspace{1cm} (14)

Note that there is no presumption that these functions are linear.

(iii) The gradients of the contribution functions are given by

$$\frac{d\Sigma_L}{dz} |_z = \alpha_L^*(z) \frac{du_L}{dz} |_z \text{ and } \frac{d\Sigma_L}{dz} |_z = \alpha_H^*(z) \frac{du_H}{dz} |_z.$$ \hspace{1cm} (15)

The coefficients $\alpha_L^*(z), \alpha_H^*(z) > 0$, for all $z$, and are differentiable functions of $z$.

(iv) The gradients of the two valence functions satisfy

$$\frac{d\mu_L}{dz} |_z = \alpha_L^{**}(z) \frac{d\Sigma_L}{dz} |_z \text{ and } \frac{d\mu_H}{dz} |_z = \alpha_H^{**}(z) \frac{d\Sigma_H}{dz} |_z,$$ \hspace{1cm} (16)

where again the coefficients $\alpha_L^{**}(z), \alpha_H^{**}(z) > 0$, for all $z$, and are differentiable functions of $z$. □

Under these assumptions, the first order equation becomes

$$\frac{d\mu_1}{dz} |_z = \left[ \alpha_L(z) \frac{du_L}{dz} |_z + \alpha_H(z) \frac{du_H}{dz} |_z \right] = 0$$ \hspace{1cm} (17)

where $\alpha_L(z), \alpha_H(z) > 0$. Since these are assumed to be differentiable functions of $z$, this equation generates the smooth one-dimensional contract curve associated with the utility functions of the activist groups. □

The solution to the first order equation will be a point on the contract curve that depends on the various coefficient functions $\{\alpha_L^*, \alpha_L^{**}, \alpha_H^*, \alpha_H^{**}\}$. Note that these various activist coefficients are left unspecified. They are determined by the response of activist groups to policy positions.

We regard Assumption 1, (i)-(iv), as natural. They posit that the utility gradient of the activist group dictates the gradient of each contribution function, which in turn gives the direction of most rapidly increasing valence for party 1.

To apply this analysis, we suppose that an economic activist, situated on the left of the economic axis, with preferred point $L = (x_l, y_l)$ has a utility function $u_L(x, y)$ based on the “ellipsoidal cost function,” so

$$u_L(x, y) = A - \left( \frac{(x-x_l)^2}{a^2} + \frac{(y-y_l)^2}{b^2} \right).$$ \hspace{1cm} (18)

10
Assuming that $a < b$ means that such an activist is more concerned with economic policy than currency issues. We also suppose that a hard currency activist with preferred point $H = (x_h, y_h)$ has a utility function $u_H(x, y)$ based on a cost function, so

$$u_H(x, y) = E - \left( \frac{(x - x_h)^2}{e^2} + \frac{(y - y_h)^2}{f^2} \right). \quad (19)$$

Assuming that $f < e$ means that such an activist is more concerned with currency policy than with standard left right economic issues. The contract curve generated by these utility functions is given by the equation

$$L \frac{d\alpha_L}{dz} + H \frac{d\alpha_H}{dz} = 0. \quad (20)$$

with $\alpha_L \geq 0, \alpha_H \geq 0$, but $\alpha_L, \alpha_H \neq (0, 0)$. Using this expression we can show that the “contract curve,” between the point $(x_l, y_l)$ and the point $(x_h, y_h)$, generated by the utility functions is given by the equation

$$\frac{(x - x_l)}{a^2} \frac{b^2}{(y - y_l)} = \frac{(x - x_h)}{e^2} \frac{f^2}{(y - y_h)}. \quad (21)$$

This can be rewritten as

$$\frac{(y - y_l)}{(x - x_l)} = \gamma_1 \frac{(y - y_h)}{(x - x_h)} \text{ where } \gamma_1 = \frac{b^2}{a^2} \frac{e^2}{f^2} > 1. \quad (22)$$

This “contract curve” between the two activist groups, centered at $L$ and $H$, is a catenary, whose curvature is determined by the “salience ratios” $(\frac{b}{a}, \frac{e}{f})$ of the utility functions of the activist groups. By Equation (17), this catenary can be interpreted as the closure of the one-dimensional locus of points given by the first order condition for maximizing the total valence $\mu_1(z_1) = \mu_L(\Sigma_L(z_1)) + \mu_H(\Sigma_H(z_1))$, generated by the contributions $(\Sigma_L, \Sigma_H)$ offered by the two groups of activists.

We therefore call this locus the activist catenary for 1. Note that while a position of candidate 1 on this catenary satisfies the first order condition for maximizing the total valence function it need not maximize vote share. In fact, the maximization of vote share requires considering the marginal electoral effect. From Theorem 1, the first order condition is given by the balance equation for 1:

$$\left[ \frac{d\epsilon_1^*}{dz_1} - z_1^* \right] + \frac{1}{2\beta} \left[ \alpha_L(z_1^*) \frac{d\alpha_L}{dz_1} + \alpha_H(z_1^*) \frac{d\alpha_H}{dz_1} \right] = 0. \quad (23)$$
The coefficient functions, \( \{\alpha_L, \alpha_H\} \), depend on the various gradient coefficients introduced under Assumption 1, and are explicitly written as functions of \( z_1^* \). The locus of points satisfying this equation is called the \textit{balance locus for 1}. It is also a one-dimensional smooth catenary, and is obtained by shifting the contract curve for the activists centered at \( L \) and \( H \) towards the weighted electoral mean of 1. Notice, for example, that if \( \alpha_H^*(z_1) \), the coefficient that determines the willingness of the currency activist group to contribute, is high, then this group will have a significant influence on the position of party 1. Obviously, maximizing vote share depends on the second order condition on the Hessian of the vote function \( V_1 \), and this will depend on the various coefficients and on \( \frac{d^2V_1}{dz^2} \). Moreover, the weighted electoral mean of 1 depends on the weighted electoral coefficients

\[
[\alpha_{i1}] = \left[ \frac{\rho_{i1}(1 - \rho_{i1})}{\sum_i(\rho_{i1}(1 - \rho_{i1}))} \right]
\]

and thus on the valence functions as well as the location of the opposition candidate. By the implicit function theorem we can write \( z_1^*(z_2) \) for the one dimensional solution to the balance equation for 1, at fixed \( z_2 \). The particular solution will depend on the coefficients, and can be found by examining the Hessian.

In the same way, if there are two activist groups for party 2, centered at \( R = (x_r, y_r) \) and \( S = (x_s, y_s) \) with utility functions based on ellipsoidal cost functions, with

\[
u_R(x, y) = G - \left( \frac{(x - x_r)^2}{g^2} + \frac{(y - y_r)^2}{h^2} \right), g < h \tag{25}\]

and
\[
u_S(x, y) = K - \left( \frac{(x - x_s)^2}{r^2} + \frac{(y - y_s)^2}{s^2} \right), r > s, \tag{26}\]

then the “contract curve” between the point \((x_r, y_r)\) and the point \((x_s, y_s)\) is given by the equation

\[
\frac{y - y_r}{x - x_r} = \gamma_2 \frac{y - y_s}{x - x_s} \tag{27}\]

where

\[
\gamma_2 = \frac{hr^2}{gs^2}. \tag{28}\]

As before, this contract curve gives the first order condition for maximizing the valence function

\[
\mu_2(z_2) = \mu_R(\Sigma_R(z_2)) + \mu_S(\Sigma_S(z_2)) \tag{29}\]
and can be identified with the activist catenary for 2, given by
\begin{equation}
\left[\alpha_R(z)\frac{du_R}{dz} + \alpha_S(z)\frac{du_S}{dz}\right] = 0,
\end{equation}
derived from the utility functions \(u_R\) and \(u_S\) from the activist groups located at \(R\) and \(S\) respectively. The locus of points on which vote share is maximized is given by the balance locus for 2:
\begin{equation}
\left[\frac{d\Sigma^*_2}{dz_2} - z_2^*\right] + \frac{1}{2\beta} \left[\alpha_R(z_2^*)\frac{du_R}{dz_2} + \alpha_S(z_2^*)\frac{du_S}{dz_2}\right] = 0.
\end{equation}
Again, this locus is obtained by shifting the activist contract curve for 2, to adjust to the electoral pull for the party. The coefficients will be determined by the second order condition on \(V_2\).

**Assumption 2**
We assume that the contribution functions, \(\Sigma_L, \Sigma_H\) are concave in \(z_1\), and the contribution functions \(\Sigma_R, \Sigma_S\) are concave in \(z_2\).

We further assume that the valences \(\mu_L, \mu_H, \mu_R, \mu_S\) are concave functions of \(\Sigma_L, \Sigma_H, \Sigma_R, \Sigma_S\) respectively.

We therefore assume that the total activist valence functions
\begin{equation}
\mu_1(z_1) = \mu_L(\Sigma_L(u_L(z_1))) + \mu_H(\Sigma_H(u_H(z_1))).
\end{equation}
and
\begin{equation}
\mu_2(z_2) = \mu_R(\Sigma_R(u_L(z_2))) + \mu_S(\Sigma_S(u_S(z_2))).
\end{equation}

are concave functions of \(z_1, z_2\) respectively. \(\Box\)

These assumptions appear natural because (i) the utility functions of the activist groups for both 1 and 2 are concave in \(z\), and (ii) the effect of contributions on activist valence can be expected to exhibit decreasing returns.

Theorem 1 uses the condition that the Hessians of these valence functions with respect to \(z_1, z_2\) respectively, have negative eigenvalues of high modulus. This gives a sufficient condition for existence of PNE.

The term balance solution is used in Definition 4 for a pair of positions \((z_1^*, z_2^*)\) satisfying both balance equations. The same idea can also be developed when there are more than two activist groups for each candidate.

As noted above we can write \(z_1^*(z_2)\) for the locus of points satisfying the balance equation for 1 at fixed \(z_2\). This balance locus given by the function \(z_1^*(z_2)\) will lie in a domain bounded by the contract curve of the activists who contribute to party 1. A similar argument gives the balance
locus $z_2^* (z_1)$ which again will be a one dimensional curve, which lie in a
domain bounded by the contract curve of the activists who contribute to 2.
The balance solution is then given by $(z_1^* (z_2^*), z_2^* (z_1^*))$. Because each balance
locus is one-dimensional, and therefore of codimension one, the solution will
generically be of dimension zero, namely a point (Schofield 2003). There
may be many zero-dimensional balance solutions, but the assumption of
sufficient concavity of the total valence functions gives a balance solution
which is a PNE. The same argument can be carried out for an arbitrary
number of parties (Schofield 2001).

4 Argentina’s Electoral Dynamics: 1989-1995

As discussed in the Introduction, prior to the election of 1989, Argentina was
under the administration of the UCR and in the grip of hyperinflation. Carlos Menem, the candidate for the opposition party, PJ, adopted a populist
platform well to the left of the electoral center on the traditional left-right
axis. Menem proposed typical redistributive policies in favor of the working
class coupled with incentives to the “productive sector” of the economy. In
contrast, the platform proposed by Angeloz focused on fiscal discipline and
a reduced role of the state. Thus, a one-dimensional policy space seems a
reasonable approximation to Argentina in 1989.\footnote{This standard, unidimensional, model of voting has been widely used in the literature.
For example, see Acemoglu and Robinson (2005), Herrera, Levine and Martinelli (2005)
and Osborne and Slivinski (1996).}

The results of section 2 suggest that there are two different cases de-
pending on the parameters of the model.

First, suppose that the convergence coefficient $c = 2\beta (1 - 2\rho) v^2$ is
bounded above by the dimension of the policy space, $w=1$. In this case,
we say that the critical condition is satisfied. If the valences are very similar
(with $|\lambda_{PJ} - \lambda_{UCR}|$ close to zero), then the vote share, $\rho$, of both parties
will be close to $1/2$, and $c$ will be close to 0. As a result, the electoral origin
will be a local equilibrium. With just two parties, Corollary 2 asserts that
this critical condition is given by $\beta \leq \beta_0$, where

$$\beta_0 = \frac{\exp(\lambda_{PJ} - \lambda_{UCR}) + 1}{2v^2 \exp(\lambda_{PJ} - \lambda_{UCR}) - 1}.$$  \hspace{1cm} (34)

Note that if $\lambda_{PJ}$ approaches $\lambda_{UCR}$, then $\beta_0$ approaches $\infty$ so the critical
condition is always satisfied.
For the sake of exposition we consider only two parties, but a similar critical condition can be obtained for an arbitrary number of parties. In fact, in 1989 three candidates contested the election. Angeloz obtained 37% of the votes, Menem 47%, and Alsogaray, a rightist candidate, 7%.

We refer the reader to Figure 1, in which we assume a distribution of voter ideal points whose mean is the electoral origin. The left-right axis is termed the “labor-capital” axis in the figure. The vertical axis may be ignored by the moment. The actual strategies of the PJ and UCR in the 1989 election are represented by the points $PJ_{1989}$ and $UCR_{1989}$, respectively. Prior to the election, we may suppose that $|\lambda_{PJ} - \lambda_{UCR}|$ was indeed close to zero. In a model without activists there would be no reason for either party to vacate the center. Notice, however, that a perturbation in the valences of the parties, so that $|\lambda_{PJ} - \lambda_{UCR}| \neq 0$, will induce a move by the low valence party away from the origin whenever $\beta > \beta_0$.

In this second situation with $\lambda_{PJ} > \lambda_{UCR}$, if both the electoral variance $\sigma^2$ and the spatial coefficient $\beta$ are large enough, then the low valence party, the UCR, should retreat from the origin, towards $UCR_{1989}$ in order to increase its vote share.

However, if the UCR cannot obtain electoral support from activists then it will lose the election. The consequence will be that both PJ and UCR should move further apart, in opposite directions away from the electoral origin, to obtain increasing support from the left activists, at $L$ (for the PJ) and from the conservative activists at $R$ (for the UCR). The vote maximizing equilibrium ($PJ_{1989}$, $UCR_{1989}$) results from these centrifugal moves to balance the attraction of the weighted electoral mean and the influence of the activists. Menem’s higher valence, gave him the electoral victory.

The point $L$ can be taken to be the preferred policy of the working class “syndical” leaders, who provided key support for Menem’s 1989 electoral victory. Because the choices of the syndical leaders were followed by a large part of the Argentinean working class, the effect of this support, represented by the valence function $\mu_L$, was pronounced. This explains why Menem’s strategy against a discredited UCR was far to the left, as indicated in Figure 1. This analysis seems to be a fairly accurate description of Argentina’s polity for the election of 1989. We now use the model to analyze the events after 1989, leading up to the 1995 election.

The main issue is whether Menem’s drastic and successful repositioning after the 1989 election can be explained by our model. Until hyperinflation was defeated, any debate regarding the optimal real exchange rate was fruitless. Thus, it was not until Menem’s Convertibility Plan stabilized the level of prices that the currency issue gained significant saliency. Because
the Convertibility Plan was successful against hyperinflation through fixing the nominal rate of exchange of the Argentinean peso in a 1-to-1 ratio to the American dollar, the currency issue naturally ended up focusing on the Convertibility Plan itself. The Convertibility Plan became most salient during the Mexican crisis, popularly known as “Tequila,” in December 1994. Because the next presidential election (in which Menem would seek his re-election) was scheduled for May 1995, the issue dominated the electoral debate. The vertical axis in Figure 1 represents the policy options in this new axis, which we refer to as the “currency dimension.”

Two groups gained from the Convertibility Plan. The European firms that won most of the privatization concessions of Argentinean companies benefited from the progressive appreciation of the peso after 1991 via the increased value in their assets and profits. Though these originated in Argentina, they were denominated in dollars. The upper middle class benefited from this policy too, since it enjoyed a consumption boom of foreign goods and the reappearance of credit after so many years of high inflation.\(^2\)

The main losers from the Convertibility Plan were perhaps those small and medium entrepreneurs, and their employees, whose firms could not overcome the difficulties associated with the appreciation of the Argentinean currency and the openness of the economy.\(^3\)

Among all groups affected by the value of the currency, the privatized companies had the greatest potential as an effective activist group. This was a consequence of their small number, their large pool of financial resources and the their lobbying power. On the other hand, any attempt at activism against the Plan by either small and medium entrepreneurs and their employees had to overcome the standard Olsonian collective action problem.

Consider again the positions \(PJ89^*\) and \(UCR89^*\) on either side of, and approximately equidistant from, the electoral origin as in Figure 1. The Figure also gives balance loci for the PJ and UCR in 1989 and 1995 that we argue can be imputed from the four different activist groups centered at \(L\), \(H\), \(R\) and \(S\).\(^4\)

\(^2\)Based on available polls, it has been estimated that the intention of vote for Menem had the following pattern –where the first percentage corresponds to the 1989 election and the second to the 1995 election. Among the voters with low to moderate income, it decreased from 63% to 59%. Among the voters of middle and upper middle income, it increased from 40% to 49% and from 38% to 47%, respectively. Finally, among the upper class voters, it increased from 13% to 42% (Gervasoni, 1997).

\(^3\)The rate of unemployment peaked at 18% in 1995, the year in which the Tequila affected the Argentinean economy.

\(^4\)A note of caution is crucial to properly interpret Figure 1. Menem certainly moved to
To apply this model developed in the previous section, consider a move by Menem along the balance locus from position $PJ^{89}$ to position $PJ^{95}$. In such case, Menem would certainly gain the support of the activists located at $H$, while losing some of the political contributions of erstwhile supporters located at $L$. While $\Sigma_L$ would fall, $\Sigma_H$ would increase. Because of the mentioned higher marginal gain of the hard currency activists, we expect $\mu_L + \mu_H$ to increase. This reasoning is reinforced by the assumption of concavity of each activist valence function, since this implies that $\frac{d\mu_L}{d\Sigma_{PJ}}$ would be positive and high, and $\frac{d\mu_L}{d\Sigma_{PJ}}$ would be negative, but of low modulus, as the $PJ$ position moves along the balance locus away from $L$.

It is our interpretation of the change from 1989 to 1995 that the increase in Menem’s exogenous valence, due to the initial success of the Convertibility Plan, together with the emergence of hard currency activist support, explains $PJ$’s policy position. The increased exogenous valence shifted the balance locus for the $PJ$ towards the origin, while the emergence of the hard currency activist group, in turn, induced $PJ$ to move down along the balance locus, to $PJ^{95}$. These effects are illustrated in Figure 1.\footnote{Seligson (2003) and Szusterman (1996) discuss the electoral platforms of $PJ$, $UCR$ and FREPASO in the 1995 election. Once the perceptions of the population on each platform is accounted for, we find no disagreement between their estimates and ours (as illustrated in Figure 1).}

Conversely, the exogenous increase in $\lambda_{PJ} - \lambda_{UCR}$ shifted the $UCR$ balance locus. Our model suggests that this change would imply an optimal position for the $UCR$ at a position such as $UCR^{95}$ in Figure 1. Indeed, the drop in $UCR$ valence led to a search for disaffected voters in the “northwest” region of the figure. A centrist position for the $UCR$, say at $UCR^{95}$, would not cause centrist voters to choose the $UCR$ with high probability (because of the higher exogenous valence of the Menem). We suggest that $(PJ^{95}, UCR^{95})$ is a local equilibrium. With the assumption of sufficient concavity, this would be a PNE.

Two candidate slates competed in the UCR 1995 primaries. The Storani-Terragno slate adopted a position similar to $UCR^{95}$ in the figure, which our model suggests is an optimal response to $PJ^{95}$. The other, the slate Massaccesi-Hernandez, adopted the position $UCR^{95}$ in the figure. By our
argument this was not a best response. Because of its low exogenous valence, severely aggravated by events between 1989 and 1995, the UCR could not win with a centrist position.

The Massacesi-Hernandez slate won the primaries, so we represent the UCR position by $UCR_{95}$. The UCR suffered a historical defeat, obtaining only 16% of the vote. Moreover, a new party, FREPASO, outperformed UCR, with 28% of the vote. Its position, denoted $FREPASO_{95}$ in Figure 1, was close to $UCR_{95}$, although somewhat to the left on the economic axis.

5 Concluding Remarks

We have presented a general model of elections and used its insights to analyze the complex Argentinean polity of the 1990’s.

The success of the Convertibility Plan in controlling hyperinflation changed three of the political variables in the Argentinean polity: (i) it critically altered the relative valence of the two main parties, (ii) it introduced a second dimension, and (iii) it created a strong activist group.

We used our model, together with these changes, to account for the seeming paradox of non-convergence of the parties to the electoral origin.

The model implies that the changes in the political variables led to new equilibrium strategies of the candidates. The higher-valence candidate adopted a position closer than previously to the electoral center, and was supported by upper middle class voters. The low-valence candidate miscalculated, and moved even closer to the electoral center and suffered a damaging defeat. Part of this defeat was due to the emergence of a third party, which adopted a position close to what we estimate should have been UCR’s optimal strategy.

In our model, the Convertibility Plan, the fundamental cause of these exogenous changes, was the result of a clever electoral strategy adopted by Menem. We have suggested elsewhere (Cataife and Schofield 2006) that the creation of the Convertibility Plan was due to the alignment of interests of three different actors: (i) Argentina’s upper-middle class (ii) money-motivated domestic politicians and (iii) the U.S. Department of Treasury (representing the interests of U.S. Government). Because politicians need to win office in order to pursue their ultimate goals, and because the upper-

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6 An appealing interpretation of the defeat of the Storani-Terragno slate is that it was due to the pressure of party activists, who are usually more interested in policy than in winning elections.
middle class provided unusual activist support, the model given here gives a framework with which to understand what happened. Elaborating the model to examine the game between activists and candidates would also involve an analysis of the desire for illicit gains on the part of the candidates, and the compatibility of these political motivations with foreign interests.

Acknowledgement: This paper is based on research supported by NSF Grant SES 024173

6 References


Figure 1: Argentinean Presidential Elections 1989-1995

Soft currency

Hard currency

Cont contract curve between economic left and pro-soft currency activists

Economic leftist indifference curve

FREPASO95

Labor

Capital

Pro-hard currency indifference curve

Contract curve between economic conservatives and pro-hard currency activists

Balance locus UCR 95

Balance locus UCR 89

Balance locus PJ 95

Balance locus PJ 89

PJ89*

PJ95*

H

UCR89*

UCR95*

UCR95**

S

L

R

Figure 1: Argentinean Presidential Elections 1989-1995